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The Independent Number of the Co-prime Power Order Graph of finite Groups

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Abstract

In relation to a given group, its vertex set $V(\beta_G)$ characterizes the co-prime power order graph of that group, denoted by G and represented by β_G. If and only if gcd $(|x|,|y|) = p^m$, where p is a prime number and m is a non-negative integer, then the vertices x and y are adjacent. This paper established to work on the graph β_G, and its induced graphs to determine the independence set and the independence number $\alpha(\beta_G)$. All of these notions focus on the order of vertex set $V(\beta_G)$ of a finite group G, and the independent set, and $\alpha(\beta_G)$ are depend on some properties of the graph β_G as the fullness of the generalized quaternion group, dihedral group, and cyclic group.

Keywords: Coprime graph $\boldsymbol{\Gamma_G}$; Co-prime power order graph $\boldsymbol{\beta_G}$; Independent set; Independent number

1. Introduction

In mathematics there are many branches studied since a long time ago. A graph is one of most an important of these branches. This branch was focused on the study of algebraic structures of the sets, semi-group, and groups. Through the previous studies we can note the relationships between the different graphs and there vertices properties like the order of vertex. At the end of the last century, some researchers focused on the coprime graph, and all of their studies was around the properties. Sattanathan introduced a coprime graph Γ_G of a group G such as any two vertices $a, b \in V(\Gamma_G)$ are adjacent if and only if their greatest common divisor is 1[1]. In [7,8,9], the researchers looked for the properties of a graph Γ_G. Moreover, the researchers generalized a graph Γ_G into a new graph θ(G) which called coprime order graph such that each two vertices a, b ∈ V(θ(G)) are adjacent if and only if $\gcd(o(a), o(b)) = 1$ *or* p, *when* p *is prime* [2-4]. In [3-10], the researchers studied some properties of graph θ(G) as completeness, connectivity, independent set, independent number, and planarity of finite groups G . Abd Rhani *et al* are introduced the relative coprime graph of a group G with respect to subgroup H and denoted by $\Gamma_{copr}(H, G)$ such that any two distinct vertices a *and* $b \in V(G)$ are adjacent if and only if the greatest common divisor of their orders is 1, a *and* $b \in H$ [11]. In [12] Norarida *et al* are introduced some properties of $\Gamma_{copr}(H, G)$ as the independent number. In [13] R M Muthulakshmi *et al* introduced the independence number of coprime graph with other properties. Hasan & Talibi are extended the graph θ(G) into β_G such that, given a prime number p and a non-negative integer n, any two vertices x and y are neighboring if and only if $\gcd(|x|, |y|) = p^n$[14]. After that, Mushatet & Talebi investigated on some properties of the graph β_G[14]. In fact, the reader can note the author worked on important properties of the graph β_G particularly the independent set and the independent number. This research has been recognized as following: they introduced the fundamental definitions, propositions, and theorems in section 2. The author introduced the independent set and the independent number in section 3. After that, the author represented the conclusion of this research.

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2. Preliminaries & Notations

This section is dedicated to introduce the mains definitions, theorems, and notations that used in this paper. The author is represented them as following:

Definition 2.1[14]: Let G be a group of order greater than 2 then G is called *ϕ-group* if its nontrivial elements' order is a prime power.

Definition 2.2[14]: A graph β_G is complete means that every pair of points in $V(\beta_G)$ are adjacent.

Theorem 2.1[14]: The completeness of a graph β_G is equivalent to the fact that G is a *ϕ-group.*

Example 2.1: Let $G = S_3 = \{e, (12), (13), (23), (123), (132)\}$, then we see that $|(12)| = |(13)| = |(23)| = 2$, and $|(123)| = |(132)| = 4 = 2^2$. Hence S_3 is a ϕ-group, so for any vertex $x \in V(\beta_{S_3})$ satisfies the condition$|x| = 2^k$, $k \in \{0,1,2\}$. It follows that any two vertices in $V(\beta_{S_3})$ are adjacent. Therefore β_{S_3} is complete graph.

Theorem 2.2[14]: A graph $\beta_{\mathbb{Z}_n}$ is complete as soon as $\mathbb{Z}_n$ is a *ϕ-group.*

Theorem 2.3[14]: A graph $\beta_{\mathbb{Z}_n}$ is complete as soon as $n = p^k$,with a prime number p, and $k \in \mathbb{Z}^+$.

Example 2.2: Let $G = \mathbb{Z}_8 = \{0, 1,2,3,4,5,6,7\}$, then we note that $|0| = 1 = 2^0$, $|1| = |3| = |5| = |7| = 8 = 2^3$, and $|2| = 4 = 2^2$. Hence $\mathbb{Z}_8$ is any two vertices in *ϕ-group*, or in any two vertices in $V(\beta_{\mathbb{Z}_8})$ are adjacent, therefore $\beta_{\mathbb{Z}_8}$ is a complete graph.

Theorem 2.4[14]: A graph β_{D_n} is complete as soon as D_n is a $\phi - group$.

Theorem 2.5[14]: A graph β_{D_n} is complete as soon as $n = p^m$, with a prime number p, and $m \in \mathbb{Z}^+$.

Example 2.3: Let

$G = D_8 = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$, then we find that $|e| = 1 = 2^0$, $|a| = |a^3| = |a^5| = |a^7| = 8 = 2^3$, $|a^2| = 4 = 2^2$, $|a^4| = 2 = 2^1$, and $|b| = |ab| = |a^2b| = \cdots = |a^7b| = 2 = 2^1$.

Therefore D_8 is a ϕ-group and any two of the vertices in $V(\beta_{D_8})$ are two adjacent ones. Hence β_{D_8} is a complete graph.

Theorem 2.6[14]: A complete graph $\beta_{Q_{4n}}$ is complete as soon as Q_{4n} is a *ϕ-group.*

Theorem 2.7[14]: A graph $\beta_{Q_{4n}}$ is complete as soon as $n = 2^m$, $m \in \mathbb{Z}^+$.

Example 2.4: Let

$G = Q_{16} = Q_{4(4)} = \{e, a, a^2, a^3, a^4, a^5, a^6, a^7, b, ab, a^2b, a^3b, a^4b, a^5b, a^6b, a^7b\}$, then we find that $|e| = 1 = 2^0$, $|a| = |a^3| = |a^5| = |a^7| = 8 = 2^3$, $|a^2| = 4 = 2^2$, $|a^4| = 2 = 2^1$, and $|b| = |ab| = |a^2b| = \cdots = |a^7b| = 2 = 2^1$. Hence Q_{16} is a ϕ-group, and any two vertices $x, y \in V(\beta_{Q_{16}})$ are adjacent. Therefore $\beta_{Q_{16}}$ is a complete graph.

Definition 2.3[5,7]: Let Γ_G be a graph with a vertex set $V(\Gamma_G)$, and $D \subseteq V(\Gamma_G)$ then the induced graph of Γ_G denoted by $\Gamma_G[D]$ with any adjacent vertices $x, y \in \Gamma_G$ they must adjacent in $\Gamma_G[D]$.

Definition 2.4[14]: A set P(G) is a subset of a group G such that $\mathrm{P(G)} = \{x \in G \| x| = p^k, for\ a\ prime\ number\ p, and\ a\ non-negative\ integer\ \alpha\}$.

Theorem 2.8[14]: The induced subgraph $\beta_G[\mathrm{G} - \mathrm{P(G)}]$ is a null graph if and only if $|G| = p^\gamma q^\alpha$, for prime numbers p, q, and $\alpha, \gamma \in \mathbb{Z}^+$.

Example 2.5: Let $G = \mathbb{Z}_{10} = \{0, 1,2,3,4,5,6,7,8,9\}$, then we note that $|0| = 1 = 2^0$, $|1| = |3| = |7| = |9| = 10 = 2 \times 5$, $|2| = |4| = |8| = 4 = 2^2$, $|6| = 5 = 5^1$ and

$|5| = 2 = 2^1$. We obtain that $\mathbb{Z}_{10} - P(\mathbb{Z}_{10}) = \{1,3,7,9\}$, any two vertices in $\beta_{\mathbb{Z}_{10}}[\mathbb{Z}_{10} - P(\mathbb{Z}_{10})]$ are not adjacent. Hence $\beta_{\mathbb{Z}_{10}}[\mathbb{Z}_{10} - P(\mathbb{Z}_{10})]$ is a null graph.

Definition 2.5[11-13]: Assume that a graph Γ_G has a vertex set $V(\Gamma_G)$ .If any two vertices a and b in D are not adjacent in Γ_G , then the nonempty subset $S \subseteq V(\Gamma_G)$ is called an independent set of graph Γ_G.

Definition 2.6[11-13]: The independent number of graph Γ_G denoted by $\alpha(\Gamma_G)$ and it forms the size of the maximum independent set of a graph Γ_G .

3. The independent number of the graph β_G of a finitey group

In this section we investigate the independent set and the independent number of the graph β_G of a finite groups. Furthermore, we investigate the independent set and the independent number of the graph $\beta_G[G - P(G)]$.

Definition 3.1: Suppose that there is a graph β_G with vertex set $V(\beta_G)$, then the subset $D \subseteq V(\beta_G)$ is called independent set of β_G if any two vertices $a\ and\ b$ in D are not adjacent.

Definition 3.2: The independent number of the graph β_G denoted by $\alpha(\beta_G)$ and it forms the size of the maximum independent set of graph β_G.

Lemma 3.1: Suppose that there is a graph β_G with $|G| = n$, and $S \subseteq V(\beta_G)$ is an independent set of β_G, when G is a $\phi - group$ then the largest independent set is $S = \{v\}$, v is an arbitrary vertex in β_G.

Proof: Assume G is a ϕ-*group*, then $G = \{x \in V(\beta_G) \mid |x| = p^m$, where p is a prime , and m is a non-negative integer$\}$. Hence β_G is a complete graph. it follows that any two vertices $u\ and\ v$ in $V(\beta_G)$ are adjacent. From elementary graph theory, we can find $S \subseteq V(\beta_G)$ is an independent set of β_G and has at least one vertex v. Now by contrary, we suppose that $S = \{u, v\} \subseteq V(\beta_G)$ is maximum independent set of $V(\beta_G)$. It follows that $u\ and\ v$ are adjacent. It is a contradiction. Hence the maximum independent set is $S = \{v\}$.

Theorem 3.2: Suppose we have a graph β_G, then $\alpha(\beta_G) = 1$ if and only if β_G is a complete graph.

Proof: Suppose that $\alpha(\beta_G) = 1$, then the order of maximum independent set of $V(\beta_G)$ is 1. It follows that any two vertices $a\ and\ b$ in $V(\beta_G)$ are adjacent. Hence β_G is a complete . Now, we suppose that β_G is a complete, then G is a $\phi - group$ [Theorem 2.1]. It follows that the largest independent set $S = \{v\} \subseteq V(\beta_G)$[Lemma 3.1]. Therefore $\alpha(\beta_G) = 1$.

Example 3.1: From Example 2.1 we obtain that any pair of vertices $x, y \in V(\beta_{S_3})$ are adjacent, then β_{S_3} is a complete graph, and the maximum independent set is $\{v\}$ such that $v \in V(\beta_{S_3})$. It follows that $\alpha(\beta_{S_3}) = 1$.

Theorem 3.3: Suppose that $G = \mathbb{Z}_n$, and we have the graph $\beta_{\mathbb{Z}_n}$, then $\alpha(\beta_{\mathbb{Z}_n}) = 1$ if and only if $n = p^k, p\ is\ a\ prime, and\ k\ is\ non-negative\ integer$.

Proof: Suppose that $\alpha(\beta_G) = 1$, then $\beta_{\mathbb{Z}_n}$ is complete [Theorem 3.2]. It follows that any $u \in \beta_{\mathbb{Z}_n}$ has a prime power order such that $|u| = p^m \backslash n$. Hence $n = p^k$.

Now we suppose that $n = p^k$, then $\beta_{\mathbb{Z}_n}$ is complete[Theorem 2.2]. Therefor $\mathbb{Z}_n$ is a $\phi - group$[Theorem 2.2]. Hence $\alpha(\beta_{\mathbb{Z}_n}) = 1$ [Lemma 3.1].

Example 3.2: From Example 2.2 we have that $\beta_{\mathbb{Z}_8}$ is a complete graph and any vertex $x \in V(\beta_{\mathbb{Z}_8})$ be adjacent other vertices, and the maximum independent set of $V(\beta_{\mathbb{Z}_8})$ has one vertex. Hence $\alpha\left(\beta_{\mathbb{Z}_8}\right) = 1$. If we take $G = \mathbb{Z}_{10}$ as Example 2.5 we have $\beta_{\mathbb{Z}_{10}}$ is not complete graph, and the maximum independent set is $\{1,3,7,9\}$. It follow that $\alpha\left(\beta_{\mathbb{Z}_8}\right) = 4$.

Theorem 3.4: Let $G = D_n$, and we have the graph β_{D_n}, then $\alpha(\beta_{D_n}) = 1$ if and only if m is a non-negative integer, p is a prime, and $n = p^m$.

Proof: Suppose that $\alpha(\beta_{D_n}) = 1$, then β_{D_n} is complete [Theorem 3.2]. It follows that each $u \in \beta_{D_n}$ has satisfied $|u| = p^m$, such that $p^m \backslash n$. It implies that $n = p^k$.

Now, we assume that $n = p^k$, then β_{D_n} is complete graph[Theorem 2.5]. It follows that D_n is a $\phi - group$ [Theorem 2.4]. Therefore $\alpha(\beta_{D_n}) = 1$[lemma 3.1].

Example 3.3: From Example 2.3 we obtain that any pair of vertices $x, y \in V(\beta_{D_3})$ are adjacent, then β_{D_3} is a complete graph, and the maximum independent set is $\{v\}$ such that $v \in V(\beta_{D_3})$. It follows that $\alpha(\beta_{D_3}) = 1$.

Theorem 3.5: Let $G = Q_{4n}$, and we have the graph $\beta_{Q_{4n}}$, then $\alpha(\beta_{Q_{4n}}) = 1$ if and only if $n = 2^m, m \; is \; non - negative \; integer$.

Proof: Suppose that $\alpha(\beta_{Q_{4n}}) = 1$, then $\beta_{Q_{4n}}$ is complete [Theorem 3.2]. It means that each vertex $u \in \beta_{Q_{4n}}$ has satisfied $|u| = 2^k$, such that $2^k \backslash n$. It implies that

$n = 2^m$. Now, we assume that $n = p^k$, then $\beta_{Q_{4n}}$ is complete graph[Theorem 2.7].

It follows that Q_{4n} is a $\phi - group$ [Theorem 2.6].

Therefore $\alpha(\beta_{Q_{4n}}) = 1$[lemma 3.1].

Example 3.4: From Example 2.4 we have that $\beta_{Q_{16}}$ is complete graph, and any vertex $u \in V(\beta_{Q_{16}})$ be adjacent other vertices, and the maximum independent set of $V(\beta_{Q_{16}})$ has one vertex . Hence $\alpha\left(\beta_{Q_{16}}\right) = 1$.

Theorem 3.6: Suppose that there is a graph β_G, and $|G| = p^\alpha q^\gamma$, then

$\alpha(\beta_G[G - P(G)] = |G - P(G)|$

Proof: Suppose that $|G| = p^\alpha q^\gamma$. It follows that $\beta_G[G - P(G)]$ is a null graph [Theorem 2.8]. This signifies that no two vertices $x, y \in \beta_G[G - P(G)]$ are contiguous. Hence $G - P(G)$ is a vertex set of $\beta_G[G - P(G)]$. Therefore $\alpha(\beta_G[G - P(G)] = |G - P(G)|$.

Example 3.5: From Example 2.5 we obtain that $\mathbb{Z}_{10} - P(\mathbb{Z}_{10}) = \{1,3,7,9\}$, and the maximum independent set equal to $\{1,3,7,9\}$. Hence $\alpha\left(\beta_{\mathbb{Z}_{10}}[\mathbb{Z}_{10} - P(\mathbb{Z}_{10})]\right) = 4$.

Theorem 3.7: Suppose that there is a graph β_G, and $|G| = {p_1}^{\alpha_1}{p_2}^{\alpha_2}{p_3}^{\alpha_3}, p_1, p_2, p_3$ are distinct prime factors, and $G - P(G) = \bigcup_{i=1}^{4} \mathcal{A}_i$ such that

$\mathcal{A}_i = \{\, x \in V(\beta_G[G - P(G)]) ||x| = {p_1}^{\alpha'_1}{p_2}^{\alpha'_2} \; either \; {p_1}^{\alpha'_1}{p_3}^{\alpha'_3} \; or \; {p_2}^{\alpha'_2}{p_3}^{\alpha'_3} \; or \; {p_1}^{\alpha'_1}{p_2}^{\alpha'_2}{p_3}^{\alpha'_3}, p_i's \; are \; a \; prime \; factors \,\}$ and $\alpha_1, \alpha_2, \alpha_3, \alpha'_1, \alpha'_1, \alpha'_1$ are positive integer numbers then

$\alpha(\beta_G[G - P(G)] = \max\{|\mathcal{A}_i|\}, i \in \{1,2,3,4\}$.

Proof: Suppose that $|G| = {p_1}^{\alpha_1}{p_2}^{\alpha_2}{p_3}^{\alpha_3}$, $G - P(G) = \bigcup_{i=1}^{4} \mathcal{A}_i$ such that $\mathcal{A}_i = \{\, x \in V(\beta_G[G - P(G)]) ||x| = {p_1}^{\alpha'_1}{p_2}^{\alpha'_2} \; either \; {p_1}^{\alpha'_1}{p_3}^{\alpha'_3} \; or \; {p_2}^{\alpha'_2}{p_3}^{\alpha'_3} \; or \; {p_1}^{\alpha'_1}{p_2}^{\alpha'_2}{p_3}^{\alpha'_3}, p_i's \; are \; a \; prime \; factors \,\}$.

It follows that each two vertices $a \; and \; b \in \mathcal{A}_i$ are not adjacent, where $i \in \{1,2,3,4\}$.

It implies that any partition $\mathcal{A}_i$ of a vertex set $V(\beta_G[G - P(G)])$ forms an independent set. Therefore $\alpha(\beta_G[G - P(G)] = \max\{|\mathcal{A}_i|\}$.

Example 3.6: Let $G = \mathbb{Z}_{30} = \mathbb{Z}_{2\times3\times5} = \{0,1, \dots ,29\}$, then

$(\mathbb{Z}_{30}) = \{x \in \mathbb{Z}_{30} ||x| = 1, or \; 2 \; or \; 3 \; or \; 5\,\} = \{0,6,10,12,15,18,20,24\}$. It follows that

$\mathbb{Z}_{30} - P(\mathbb{Z}_{30}) = \{1,2,3,4,5,7,8,9,11,13,14,16,17,19,21,22,23,25,26,27,28,29\}$, and we obtain four partitions of $\mathbb{Z}_{30} - P(\mathbb{Z}_{30})$ regard to the order of element as following:

$W_{1,2} = \{x \in \mathbb{Z}_{30} ||x| = 2 \times 3\} = \{5,25\}$,

$W_{1,3} = \{x \in \mathbb{Z}_{30} \| x| = 2 \times 5\} = \{3, 9, 21, 27\}$,

$W_{2,3} = \{x \in \mathbb{Z}_{30} \| x| = 3 \times 5\} = \{2, 4, 8, 14, 16, 22, 26, 28\}$, and

$W_{1,2,3} = \{x \in \mathbb{Z}_{30} \| x| = 2 \times 3 \times 5\} = \{1, 7, 11, 13, 17, 19, 23, 29\}$.

The maximum independent sets are $W_{2,3}$, and $W_{1,2,3}$. Hence $\alpha(\beta_{\mathbb{Z}_{30}}[\mathbb{Z}_{30} - P(\mathbb{Z}_{30})]) = 8$.

Theorem 3.8: Suppose that there is a graph β_G, and $|G| = {p_1}^{\alpha_1}{p_2}^{\alpha_2} \dots {p_k}^{\alpha_k}, p_1, p_2, \dots, p_k$ are distinct prime factors, and $G - P(G) = \bigcup_{i=1}^{k} \mathcal{A}_i$ such that

$\mathcal{A}_i = \{ x \in V(\beta_G[G - P(G)]) \| x| = {p_1}^{\alpha'_1}{p_2}^{\alpha'_2} \dots {p_r}^{\alpha'_k}, p_i's \text{ are a prime factors} \}$ and $\alpha_1, \alpha_2, \dots, \alpha_k, \alpha'_1, \alpha'_2, \dots, \alpha'_r$ are positive integer numbers, $r \geq 2$ then

$\alpha(\beta_G[G - P(G)]) = \max\{|\mathcal{A}_i|\}, i \in \{1, 2, \dots, k\}$.

Proof: Suppose that $|G| = {p_1}^{\alpha_1}{p_2}^{\alpha_2} \dots {p_k}^{\alpha_k}, p_1, p_2, \dots, p_k$ are distinct prime factors, and $G - P(G) = \bigcup_{i=1}^{k} \mathcal{A}_i$ such that

$\mathcal{A}_i = \{ x \in V(\beta_G[G - P(G)]) \| x| = {p_1}^{\alpha'_1}{p_2}^{\alpha'_2} \dots {p_r}^{\alpha'_k}, p_i's \text{ are a prime factors} \}$ and $\alpha_1, \alpha_2, \dots, \alpha_k, \alpha'_1, \alpha'_2, \dots, \alpha'_r$ are positive integer numbers, $r \geq 2$, then any two vertices $u \text{ and } v \in \mathcal{A}_i$ are not adjacent , such that $i \in \{1, 2, \dots, k\}$. It follows that any partition $\mathcal{A}_i$ of a vertex set $V(\beta_G[G - P(G)])$ forms an independent set.

Therefore $\alpha(\beta_G[G - P(G)] = \max\{|\mathcal{A}_i|\}$.

4. Conclusion

In this paper, we deduced certain characteristics of the graph β_G such that, when G is a *ϕ-group*, the biggest independent set of size one and the independent number of a graph β_G is 1 if and only if β_G is complete graph. In particular, we concluded that $\alpha(\beta_{\mathbb{Z}_n}) = 1$ if and only if $n = p^k$, $\alpha(\beta_{D_n}) = 1$ if and only if $n = p^m$, and $\alpha(\beta_{Q_{4n}}) = 1$ if and only if $n = 2^m$. Furthermore, we concluded that

$\alpha(\beta_G[G - P(G)] = |G - P(G)|$ if $|G| = p^\alpha q^\gamma$, $\alpha(\beta_G[G - P(G)] = \max\{|\mathcal{A}_i|\}$, when $|G| = {p_1}^{\alpha_1}{p_2}^{\alpha_2}{p_3}^{\alpha_3}$ such that $\mathcal{A}_i = \{ x \in V(\beta_G[G - P(G)]) \| x| = {p_1}^{\alpha'_1}{p_2}^{\alpha'_2} \text{ either } {p_1}^{\alpha'_1}{p_3}^{\alpha'_3} \text{ or } {p_2}^{\alpha'_2}{p_3}^{\alpha'_3} \text{ or } {p_1}^{\alpha'_1}{p_2}^{\alpha'_2}{p_3}^{\alpha'_3}, p_i's \text{ are a prime}$

$\text{factors} \}$, and $\alpha(\beta_G[G - P(G)] = \max\{|\mathcal{A}_i|\}, i \in \{1, 2, \dots, k\}$ when $G - P(G) = \bigcup_{i=1}^{k} \mathcal{A}_i$,

and $\mathcal{A}_i = \{ x \in V(\beta_G[G - P(G)]) \| x| = {p_1}^{\alpha'_1}{p_2}^{\alpha'_2} \dots {p_r}^{\alpha'_k}, p_i's \text{ are a prime factors} \}$, $|G| = {p_1}^{\alpha_1}{p_2}^{\alpha_2} \dots {p_k}^{\alpha_k}$.

Compliance with ethical standards

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No conflict of interest to be disclosed.

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