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Higher Reverse α-Derivations of Semi-prime $\Gamma-$ Ring M

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Abstract

Suppose that $\alpha = (\alpha_i)_{i \in N}$ is a family of endomorphism of M and that M is a 2-torsion free semi-prime Γ-ring. Suppose that, $d = (d_i)_{i \in N}$ collection of flavor mappings of M , such that $d_0 = id_M$. The concept of higher reverse α-derivations of semi-prime Γ-rings is introduced and studied in this work. Such related concepts are also in our view as Jordan triple higher reverse α-derivations and Jordan higher reverse α-derivations. In relation to these mappings, there are several fundamental characteristics and structural findings that are obtained. Specifically, we consider their action on semiprime rings of Γ and obtain a variety of structural conclusions that generalize classical conclusions on derivations and higher derivations in ring theory.

Keywords: Semi-prime Γ-ring; Derivation; α-derivation; Higher derivation; Forward derivation; Jordan higher reverse alpha-derivations

1. Introduction

Nobusawa introduced the first Γ-ring which is generalization of the classical concept of a ring. This idea was later marginally undermined by Barnes, with the important algebraic structure being maintained [1-3]. A Γ- ring is in this sense a pairing of additive abelian groups M and Γ along with a mapping $M \times \Gamma \times M \to M$ to M, (s, γ t) defined by $(s, \gamma, t) \mapsto s\gamma t$, satisfying suitable distributive laws and the associativity condition $(s \gamma t) \beta r = s \gamma (t \beta r)$ and $(s\ t)\gamma\ r = s \gamma r + t \gamma r$, $s (\gamma + \beta)t = s\gamma t + s\beta t$ and , for all s, t, r ∈ M and γ, β ∈ Γ [4-6]. A Γ-ring M is called 2-torsion free if 2s = 0 implies s = 0. Moreover, M is called prime if s Γ M t = 0 implies, s = 0 or t = 0. It is called semi-prime if s Γ M s = 0 implies s = 0. Every ring is a Γ -ring, of course, but not all rings are necessarily Γ -rings [7-9].

The concept of derivations is significant in the theory of structure of rings and Γ-rings. To generalize the classical theory of derivations in rings, Jing defined derivations and Jordan derivations on Γ-rings [10-12]. Subsequently, a number of authors investigated different generalizations including higher derivations and Jordan higher derivations [13-15]. Chang coined the theory of derivations in rings and examined their algebraic characteristics [16-18]. Later workers generalized these concepts to Γ-rings and studied their behavior in various eponymous conditions. The concept of reverse derivations on rings, introduced by Bresar and Vukman, gave a dual variant of classical derivations and led to the new perspectives in the theory of derivations [7]. It was later developed by Majeed and Salih, who added higher derivations and Jordan higher derivations on Γ-rings along with a number of generalizations and structural properties [8,9]. Having these developments in mind, we generalize the idea of reverse derivations to semi-prime -negative Γ-rings [18-20]. Precisely, we construct higher reverse Γ-rings derivations, Jordan higher reverse Γ-rings derivations and Jordan triple higher reverse Γ-rings M derivations. In this paper, M refers to Semi prime Γ-ring (a 2 torsion free).

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2. Reverse α - derivation of $\Gamma - ring\ M$

2.1. Definition

Agreement α is an endomorphism mapping of $\Gamma - ring\ M$. A addition map $d: M \longrightarrow M$ named opposite $\alpha - derivation$ of M when

$$d\,(s\gamma\, t) = d(t)\gamma\, \alpha(s) + s\,\gamma\, d(t) \qquad (1)$$

d be named Jordan opposite $\alpha - derivation$ of M

$$\text{Unknown } d(s\,\gamma s) = d(s)\gamma\, \alpha(s) + s\,\gamma d(s) \qquad (2)$$

Hence d named Jordan triple opposite $\alpha - derivation\ of\ M$

$$\text{when } d\,(s\gamma\, t\,\beta\, s) = d(s)\,\beta\alpha(s)\,\gamma\alpha(t) + s\beta d\,(t)\,\gamma\,\alpha(s) + s\beta t d(s) \qquad (3)$$

Let us show examples of reverse α-derivation on Γ-ring .

2.2. Example

Assume R a ring, the additive abelian groups

$M=\left\{\begin{pmatrix} s & t \\ 0 & p \end{pmatrix} : \text{such that } s, t, p \in R\right\}$ and $\Gamma=\left\{\begin{pmatrix} 0 & 0 \\ 0 & k \end{pmatrix}, k \in \Gamma\right\}$

This means M is a Γ-ring. We defined $\alpha : M \longrightarrow M$ by:

$\alpha\left(\begin{pmatrix} s & t \\ 0 & p \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & p \end{pmatrix}, \forall \begin{pmatrix} s & t \\ 0 & p \end{pmatrix} \in M$. This means α is endomorphism of M. Suppose d: $M \longrightarrow M$ defined by $d\left(\begin{pmatrix} s & t \\ 0 & p \end{pmatrix}\right) = \begin{pmatrix} s & 0 \\ 0 & 0 \end{pmatrix}, \forall \begin{pmatrix} s & t \\ 0 & p \end{pmatrix} \in M$, so d means reverse α-derivation.

2.3. Example

Assume R defined on M . We stand for $M' = M \oplus M$, $\Gamma' = \Gamma \oplus \Gamma$ considering ⊕ is sum of M and Γ, the endomorphism mapping α′ on M′ defined by α′((s,t)) = (α(s),α(t)). Also, define an addition mapping d′ on M′ by d′((s,t)) = (d(s),0) . So d′ is reveres α′-derivation on M′.

3. Higher Reverse α-Derivation of Γ-ring M

Let us begin the following concept:

3.1. Definition

Suppose $\alpha = (\alpha_i)_{i \in N}$ collection of endomorphism mapping Γ-ring M, such that

$\alpha_0 = id_M$, $d = (d_i)_{i \in N}$ collection of flavor mappings of M , such that $d_0 = id_M$ then d be named a upper opposite $\alpha - derivation$ when

$$d_n\,(s\gamma\, t) = \Sigma_{i+j=n}\, d_i\,(t)\,\gamma\,\alpha_i\,(d_j(s)) \qquad (1)$$

This named a Jordan higher reverse α-derivation of M when we have

$$d_n\,(s\gamma s) = \Sigma_{i+j=n}\, d_i\,(s)\,\gamma\,\alpha_i\,(d_j(s)) \qquad (2)$$

Hence d is named Jordan tripartite higher opposite $\alpha - derivation\ of\ M$ when

$$d_n(s\,\gamma t\,\beta s\,) = \sum_{i+j+l=n} d_i\,(s)\beta\alpha_i\,\left(d_j(t)\right)\gamma\alpha_i\,(\alpha_j\,\,(d_l\,(s))) \qquad (3)$$

Let us notice an example on higher reverse α-derivation of Γ-ring.

3.2. Example

Assume d = $(d_i)_{i\in N}$ collection of higher opposite $\alpha - derivation\ of\ M$. Set $M' = M \oplus M\ and\ \Gamma' = \Gamma \oplus \Gamma where \oplus$ is direct Sum of M and Γ, the family of endomorphism mapping $\alpha' = (\alpha'_i)_{i\in N}$ is given α'_n ((s,t)) = $(\alpha_n(s),\alpha_n(t))$, (s,t) $\in$ M′ . Suppose d′ = $(d'_i)_{i\in N}$ defined on M′ by d'_n ((s,t)) = $(d_n(s),o)$, so d'_n is higher reverse α'-derivation on M′.

4. Key Marks

In this part we will current and learning some properties of upper reverse α-derivation on Γ-ring M which make us able to prove several theorem concerning this definition as orthogonally.

4.1. Lemma

Suppose $\alpha = (\alpha_i)_{i\in N}$ collection of endomorphism of Γ-ring M, d = $(d_i)_{i\in N}$ Jordan higher reverse $\alpha - derivations\ on\ M$. Then the resulting statements hold :

(i) $d_n\,(\,s\,\gamma \mathrm{t}\, +\, \mathrm{t}\,\gamma s\,) = \sum_{i+j=n} d_i(t)\,\gamma\,\alpha_i\,(d_j(s)) + d_j\,(s)\,\gamma\,\alpha_i((d_j\,(t)))$

(ii) In particular α be into mapping then :

$$d_n(\,s\,\gamma t\beta s\, + s\beta\,t\gamma s\,) = \sum_{i+j+l=n} d_i\,(s)\beta\alpha_i\,\left(d_j(t)\right)\gamma\,\alpha_i\,(\alpha_j\,(d_l\,(s)))$$

$$+ d_i\,(s)\gamma\alpha_i\,(d_j\,(s))\,\beta\alpha_i\,(\alpha_j\,(d_l\,(s)))\,.$$

(iii) When M be a 2-torsion free then :

$d_n\,(\,s\,\gamma \mathrm{t}\,\gamma s\,) = \sum_{i+j+l=n} d_i\,(s)\gamma\alpha_i\,(d_j(t))\gamma\,\alpha_i\,(\alpha_j\,(d_l\,(s)))$.

(iv) $d_n\,(\,s\,\gamma \mathrm{t}\,\beta\,s\, + s\gamma t\,\beta s) = \sum_{i+j+l=n} d_i\,(s)\,\beta\alpha_i\,(d_j(\mathrm{t}))\,\gamma\,\alpha_i\,(\alpha_j\,(d_l\,(s)))$

$$+ d_i\,(s)\,\beta\alpha_i\,\left(d_j(\mathrm{t})\right)\gamma\,\alpha_i\,(\alpha_j\,(d_l\,(s)))\,.$$

(v) $d_n\,(\,s\gamma \mathrm{t}\gamma r\, +\, \mathrm{r}\gamma \mathrm{t}\gamma s\,) = \sum_{i+j+l=n} d_i\,(\mathrm{r})\gamma\alpha_i\,(d_j(\mathrm{t}))\,\gamma\,\alpha_i\,(\alpha_j\,(d_l\,(s))) + d_i\,(s)\gamma\alpha_i\,\left(d_j(\mathrm{t})\right)\gamma\,\alpha_i\,(\alpha_j\,(d_l\,(s)))$.

Proof :

(i) Since

$$d_n\,((s\, + \mathrm{t})\,\gamma(s + \,\mathrm{t})\,) = \sum_{i+j+l=n} d_i\,((s\, + t))\,\gamma\alpha_i\,(d_j(s\, + t))$$

$$= \sum_{i+j=n} (d_i(s\,) + d_i(t))\gamma\alpha_i\,(d_j(s)\, + d_j(t))$$

$$= \sum_{i+j=n} (d_i(s\,) + d_i(t))\gamma\alpha_i\,(d_j(s)\, + \alpha_i(d_j(t))$$

$$= \sum_{i+j=n} d_i(s\,)\gamma\alpha_i(\,d_j(s)) + d_i(s)\gamma\alpha_i(d_j(t)) + \,d_i(t)\gamma\alpha_i\,(d_j(s)) + (t)\gamma\alpha_i(d_j(t)) \qquad (1)$$

Now we have

$d_n((s+t)\gamma(s+t)) = d_n(s\gamma s + s\gamma t + t\gamma s + t\gamma t)$

$d_n((s+t)\gamma(s+t)) = d_n(s\gamma s) + d_n(s\gamma t + t\gamma s) + d_n(t\gamma t) = \sum_{i+j=n} d_i(s)\gamma\alpha_i(d_j(s)) + d_n(s\gamma t + t\gamma s) + \sum_{i+j=n} d_i(t)\gamma\alpha_i(d_j(t))$ (2)

By comparing (1) and (2) results

$d_n(s\gamma t + t\gamma s) = \Sigma_{i+j=n} d_i(t)\gamma\alpha_i(d_j(s)) + d_i(s)\gamma\alpha_i(d_j(t))$

(ii) Replace $s\beta t + t\beta s$ for t in (i) we got:

$d_n(s\gamma(s\beta t + t\beta s) + (s\beta t + t\beta s)\gamma s) = \Sigma_{i+j=n} d_i(s\beta t + t\beta s)\gamma\alpha_i(d_j(s)) + \Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(s\beta t + t\beta s)) = \Sigma_{i+j=n} d_i(s\beta t)\gamma\alpha_i(d_j(s)) + d_i(t\beta s)\gamma\alpha_i(d_j(s)) + \Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(s\beta t)) + d_i(s)\gamma\alpha_i(d_j(t\beta s)) = \Sigma_{i+j=n} d_i(t)\beta\alpha_i(d_j(s))\gamma\alpha_i(d_j(s)) + d_i(s)\beta\alpha_i(d_j(t)\gamma\alpha_i(d_j(s)) + \Sigma_{i+j+l=n} d_i(s)\gamma\alpha_i(d_j(t))\beta\alpha_j(d_l(s)) + d_i(s)\gamma\alpha_i(d_j(s))\beta\alpha_j(d_l(t))$

Replace $d_j(s)$ by $\alpha_j(d_\ell(a))$ in some terms of eq. that obtain

$= \Sigma_{i+j=n} d_i(t)\beta\alpha_i(d_j(s))\gamma\alpha_i(d_j(s)) + \Sigma_{i+j+l=n} d_i(s)\beta\alpha_i(d_j(t)\gamma\alpha_i(\alpha_j(d_l(s)) +$

$\Sigma_{i+j+l=n} d_i(s)\gamma\alpha_i(d_j(t))\beta\alpha_i(\alpha_j(d_l(s)) + \Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(s))\beta\alpha_i(d_j(t))$ (1)

On the other hand we have :

$d_n(s\gamma(s\beta t + t\beta s) + (s\beta t + t\beta s)\gamma s) = d_n(s\gamma s\beta t + s\gamma t\beta s + s\beta t\gamma s + t\beta s\gamma s)$

$= d_n(s\gamma s\beta t) + d_n(s\gamma t\beta s + s\beta t\gamma s) + d_n(t\beta s\gamma s) = \Sigma_{i+j=n} d_i(s\beta t)\gamma\alpha_i(d_j(s)) + \Sigma_{i+j=n} d_i(s\gamma s)\beta\alpha_i(d_j(t)) + d_n(s\gamma t\beta s + s\beta t\gamma s) = \Sigma_{i+j=n} d_i(t)\beta\alpha_i(d_j(s))\gamma\alpha_i(d_j(s)) + \Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(s))\beta\alpha_i(d_j(t)) + d_n(s\gamma t\beta s + s\beta t\gamma s)$ (2)

Since to mapping then we can replace $d_j(s)$ by $\alpha_j(d_\ell(a))$ and compare (1) and (2) we get:

$d_n(s\gamma t\beta s + s\beta t\gamma s) = \Sigma_{i+j+l=n} d_i(s)\beta\alpha_i(d_j(t)\gamma\alpha_i(\alpha_j(d_l(s)) + \Sigma_{i+j+l=n} d_i(s)\gamma\alpha_i(d_j(t))\beta\alpha_i(\alpha_j(d_l(s))$

(iii) Replace γ for β in (ii) we get:

$d_n(s\gamma t\gamma s + s\gamma t\gamma s) = d_n(s\gamma t\gamma s) + d_n(s\gamma t\gamma s) = 2d_n(s\gamma t\gamma s) = 2\,\Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(s\gamma t)) = 2\,\Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(t)\gamma\alpha_j(d_l(s)) = 2\,\Sigma_{i+j=n} d_i(s)\gamma\alpha_i(d_j(t)\gamma\alpha_i(\alpha_j(d_l(s))$

As M stay 2-torsion allowed now

$d_n(s\gamma t\gamma s) = \sum_{i+j+l=n} d_i(s)\gamma\alpha_i(d_j(t))\gamma\,\alpha_i(\alpha_j(d_l(s)))$

(iv) Replace $s + r$ for s into definition (3.1) (3) we obtain:

$$d_n((s+r))\gamma t\beta(s+r)) = \sum_{i+j+l=n} d_i(s+r)\beta\alpha_i(d_i(t))\gamma\,\alpha_i(\alpha_j(d_l(s+r))$$

$$= \sum_{i+j+l=n} (d_i(s) + d_i(r))\beta\alpha_i(d_i(t))\gamma(\alpha_i(\alpha_j(d_l(s))) + ((\alpha_j(d_l(r)))$$

Thus

$$d_n\left((s+r)\right)\alpha\, \mathrm{t}\beta(s+r)) = \sum_{i+j+l=n} d_i\,(s)\,\beta\alpha_i\,(d_i(t))\,\gamma\ \alpha_i\,(\alpha_j\,(d_l\,(s))) + d_i\,(s)\beta\alpha_i\,(d_i(t))\gamma\ \alpha_i\,(\alpha_j\,(d_l\,(\mathrm{r}))) + d_i(r)\,\beta\alpha_i\,(d_i(t))\gamma\ \alpha_i\,(\alpha_j\,(d_l\,(s))) + d_i\,(r)\,\beta\alpha_i\,(d_i(t))\,\gamma\ \alpha_i\,(\alpha_j\,(d_l\,(\mathrm{r}))\,) \quad (1)$$

The other side is :

d_n ((s+r)) γ tβ(s+r)) = d_n (sγ tβs +sγ tβr +rγ tβs) =d_n (sγ tβs) + d_n (rγ tβr) + d_n (sγ tβr + rγ tβs) = $\Sigma_{i+j=n}$ d_i (s) βα_i (d_i sγt)) + d_i (r) βα_i(d_i (r) βα_i d_i (rγbt)) + d_n (sγ tβr + rα tβs =$\sum_{i+j+l=n}\ d_i\,(s)\,\beta\alpha_i\,(d_i(t))\gamma\ \alpha_i\,\left(\alpha_j\,(d_l\,(s))\right) +$ $\sum_{i+j+l=n}\ d_i\,(\mathrm{r})\,\beta\alpha_i\,\left(d_j(t)\right)\ \alpha_i\,\left(\alpha_j\,(d_l\,(\mathrm{r}))\right) + d_n\,(s\gamma t\beta r +\ r\gamma t\beta s\,)$
(2)

By compare (1) and (2) we obtain:

d_n ((sγtβr + rγ tβs) = $\Sigma_{i+j+l=n}$ d_i (r) βα_i(d_i (t))) γα_i (α_j(d_l(s))) + $\Sigma_{i+j+l=n}$ d_i (s) βα_i (d_j(t)) γα_i (α_j (d_l (r)))

(v) Replacing β by γ in (iv) we find require result.

5. Conclusion

The concept of higher reverse 2-torsion free semi-prime α-derivation Γ -rings was explored in this paper. The paper generalizes the classical theory of derivations and higher derivations in rings into a more general allegorical structure. The higher reverse α-derivations, Jordan higher reverse α-derivations and higher reverse α-derivations were defined. These concepts provide natural generalizations of reverse derivations and allow a deeper understanding of additive mappings in Γ-rings. Several structural properties of these mappings were established. In particular, we derived identities and relations that describe the behavior of Jordan higher reverse α-derivations and their interaction with the algebraic structure of semi-prime Γ-rings. These findings show that the setting of Γ-rings can have many of the same qualities as derivations in classical ring theory.

The obtained results contribute to the development of derivation theory in generalized algebraic systems and open the possibility for further research on generalized derivations, higher mappings, and related operator structures in Γ-rings and other algebraic frameworks.

Future work may focus on studying these derivations on prime Γ-rings, with involution, or investigating their connections with generalized derivations and automorphism structures**.**

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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